Performance Optimization for Sorting Processes

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One method of measuring the performance of a sorting process is by comparing the percentage of good parts that are erroneously rejected while maintaining an acceptable percentage of erroneously accepted rejects (rejects in the good bin).

For example, consider the following 2 processes:

Process	Erroneously Accepted	Erroneously Rejected
1	14 ppm	2.2%
2	12 ppm	4.5%

Typically (unless 12 ppm is a requirement) process 1 would be considered to have a better performance than process 2, since process 2 falsely rejects about twice as many good parts while only improving the percentage of erroneously accepted reject parts by only 2 ppm.

By this method, the performance of a sorting process is highly dependent on the sorting tolerance settings with respect to the actual dimension tolerance.

It can be shown (Appendix A) that if the sorting tolerance is the same as the dimension tolerance, then the % erroneously rejected parts and % erroneously accepted parts are well within an order of magnitude.

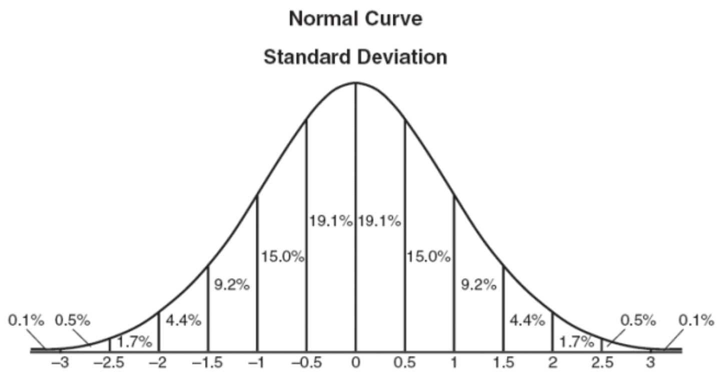
This paper describes how the performance of any sorting process can be greatly improved by modifying the sorting tolerance settings.

A method for establishing the optimal tolerance settings is also described.

The Normal Distribution Curve

We begin with a brief explanation of the Normal Distribution Curve, which applies to most manufacturing processes, including the gage itself.

Consider measuring a dimension on a significant percentage of parts within a given lot, then calculate the mean (μ) and standard deviation (σ) of all the measurements. Further assume that the measuring gage has zero error for now (See Appendix C).



The area bound by the curve and the x-axis within a range of standard deviations represents the percentage of measurements that will fall within this range. The origin of the x-axis represents the mean value for the lot (μ).

For example, 68.2% (15+19.1+19.1+15) will be between -1 and +1 standard deviations from the mean. Similarly, over 99% will be between -3 and +3 standard deviations (only 0.2% outside the range)

Of course these numbers are theoretical and will never be exact, however they give us a way to predict how many parts will be passed or rejected in a sorting process.

For the purposes of this example, we assume a centered 3-sigma process such that over 99% of the parts will be within the tolerance range.

Next we select a dimension that calls for 0.060” +/-0.003”

Then $\sigma = 0.003"/3$
$\sigma = 0.001"$

Now it’s easy to convert between σ’s and physical units.
For example 0.0017” = 1.7 σ

Next we measure several parts, each at random orientations 100 times or so, then calculate the standard deviation for the gage σ_g as described in Appendix B.

Let's assume that σ_g = 0.0002".

Now we can say with some certainty that over 99% of the measurements will be within +/-3 σ_g, or +/-0.0006" from the true dimension.

In order to reduce the percentage of out of tolerance product accepted (erroneously accepted rejects), we need to reduce the tolerance range for the sorting process. This will also increase the percentage of erroneously sorted good parts.

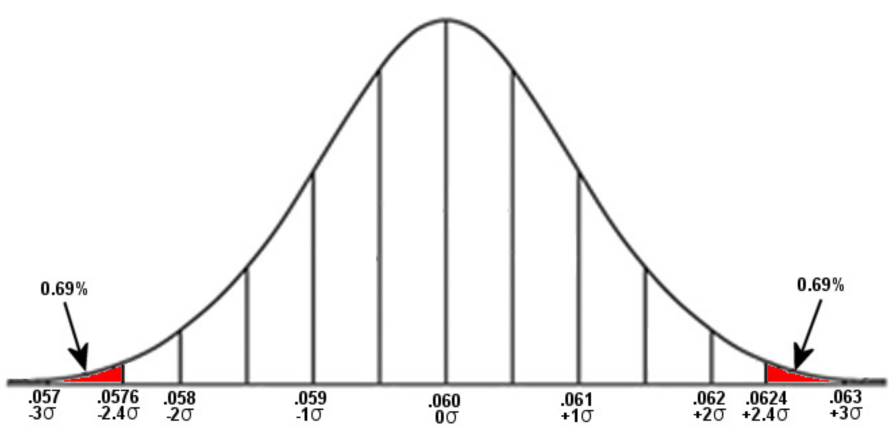
Let's examine what happens when the tolerance range is reduced by the gage's repeatability (6 σ_g), or 0.0012". This represents 20% of the original tolerance range.

At first glance, it appears that 20% of good product could be falsely rejected, however this is far from true.

The new tolerance range is 0.006" – 0.0012" = 0.0048", or 0.060" +/-0.0024"

Converting to σ: 0.0024" = 2.4 σ

Tolerance Reduced to .060" +/-0.0024"



The percentage of parts represented by the shaded area under the curve between 2.4 σ and 3 σ is the ambiguity area, where good parts could be falsely rejected.

From the z-table below, the percentage of parts to the right of 2.4 σ is 0.82%, while the percentage of parts over 3 σ is 0.13%. Therefore each shaded area represents about 0.69% of the parts.

|z|	x.x0	x.x1	x.x2	x.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
3	0.00135									
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.002	0.0019
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.003	0.0029	0.0028	0.0027	0.0026
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.004	0.0039	0.0038	0.0037	0.0036
2.5	0.0062	0.006	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.4	0.0082	0.008	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.011
2.1	0.0179	0.0174	0.017	0.0166	0.0162	0.0158	0.0154	0.015	0.0146	0.0143
2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.025	0.0244	0.0239	0.0233
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.5	0.0668	0.0655	0.0643	0.063	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.102	0.1003	0.0985
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.123	0.121	0.119	0.117
1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.166	0.1635	0.1611
0.8	0.2119	0.209	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.7	0.242	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2206	0.2177	0.2148
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.5	0.3085	0.305	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.281	0.2776
0.4	0.3446	0.3409	0.3372	0.3336	0.33	0.3264	0.3228	0.3192	0.3156	0.3121
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.352	0.3483
0.2	0.4207	0.4168	0.4129	0.409	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0	0.5	0.496	0.492	0.488	0.484	0.4801	0.4761	0.4721	0.4681	0.4641

z-table

The z-values can also be calculated mathematically using the equation:

$$z\ value = \frac{1 - \mathrm{erf}\left(\frac{z}{\sqrt{2}}\right)}{2}$$

where erf( ) is the error function.

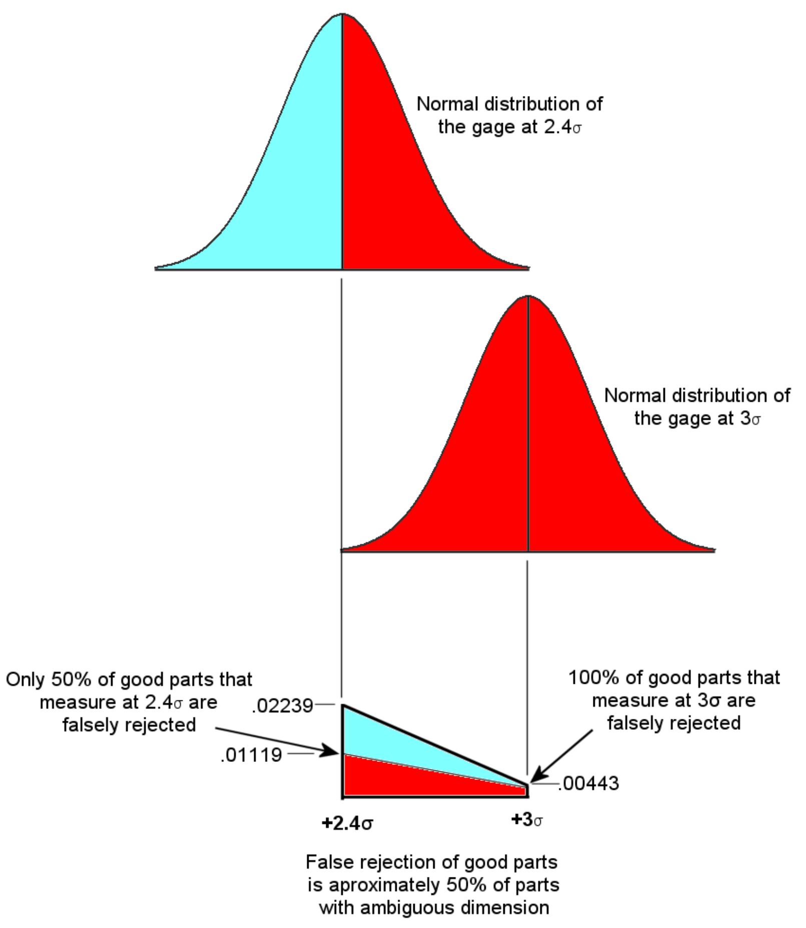
Now let's take a closer look at each shaded area, shown between 2.4 σ and 3 σ in the diagram below. The ambiguity area has been approximated by a trapezoid to keep the calculations simple.

Shown directly above the ambiguity area are the normal distribution curves for the gage, shown to scale with respect to the trapezoid. Recall that the gage itself had a standard deviation of 0.0002", therefore half the graph (3 σ = 0.0006") corresponds to the entire ambiguity width.

The normal distribution of the gage is shown applied when the actual part dimension is at 2.4 σ and at 3 σ. Some of the parts in this range will be falsely rejected, but not all.

Parts that measure close to 3 σ will be rejected almost 100% of the time, while parts that measure close to 2.4 σ will only be rejected 50% of the time because the gage will correctly sort them as good the other 50% of the time.

The red area of the trapezoid represents the percentage of parts that will falsely be rejected, or about 50% of the entire ambiguity area.



False rejection of good parts is aproximately 50% of parts with ambiguous dimension

The approximate false reject area (lower red trapezoid) can be calculated as follows:

$$\% \, false \, rejects = \frac{P(z1)/2 + P(z2)}{2} * \, \Delta z$$

where $P(z) = \frac{1}{\sqrt{2\pi}} * e^{-\frac{z^2}{2}}$

For our example:
z1 = 2.4
z2 = 3.0
Δz = 0.6
P(z1) = 0.02239
P(z2) = 0.00443

Then % False Reject Area = 0.46%

There are two such areas of ambiguity, so the total percentage of good parts erroneously rejected is about 0.92% due to the reduced tolerance range, while still guaranteeing almost zero bad parts accepted.

The very few (2 x 0.00135 x 0.00135 = 3.6 ppm) erroneously accepted parts will likely be extremely close to either tolerance limit.

Compare these results with those for full tolerance limits from Appendix A:
0.108% erroneously rejected and 0.052% erroneously accepted.

Process	Erroneously Accepted	Erroneously Rejected
Full Tolerance	0.052%	0.108%
Truncated Tolerance	0.00036%	0.92%

There is clearly a significant reduction of % erroneously accepted parts (144 times less) at the expense of a relatively small increase in % erroneously rejected parts (only 9 times more).

Less capable processes

Now that the basic principle is understood, let's analyze a less capable process.

Let's assume this time that the manufacturing process can only produce 95% of parts within the required tolerance. This is a 2-sigma process.

As with the previous example, a dimension of .060" +/-0.003" would yield:

σ = .003"/2
σ = .0015"

The gage would have the same characteristics, so that the tolerance range still needs to be reduced by 0.0012", or 20% as before.

The new tolerance range remains 0.060" +/-0.0024"

But converting to σ: 0.0024" = 1.6 σ

The ambiguity range is now between 1.6 σ and 2 σ

From the z-table: 0.0548 – 0.0228 = 0.032, or 3.2%

In this case:
z1 = 1.6
z2 = 2.0
Δz = 0.4
P(z1) = 0.1109
P(z2) = 0.0540

Then % False Reject Area = 2.19%

Multiplying by 2 to account for the other limit yields a total of 4.38% false rejects.

The falsely accepted parts can be calculated to be 2 x 0.0228 x 0.0013 = 59 ppm, as compared to the original 4.5% reject content of the unsorted lot.

Consider that 59 ppm is about the percentage of rejects allowed by a 4-sigma process. In this case, operating the sorting process with reduced tolerances provides 4-sigma quality delivery from a 2-sigma capable process, with less than 5% false rejects.

Reducing the reject content in the good bin – Optimal tolerance range

If 59 ppm rejects in the good bin is not acceptable, this can be reduced by further truncating the tolerance range.

The percentage of reject parts sorted as good can be calculated as follows:

$$\%Erroneously\ Accepted = 2 * zvalue1 * zvalue2$$

Rearranging: $$zvalue2 = \frac{\%Erroneously\ Accepted}{2*zvalue1}$$

where:
z-value1 is the z-value for the capability of the process, in this example z=2
z-value2 is the z-value required for the gage.
The constant "2" is needed to account for the opposite tolerance limit.

Let's say that we are looking for a maximum of 14 ppm rejects.

Then: $zvalue2 = \frac{0.000014}{2*0.0228}$ = 0.000307

The z-table covers only up to z=3, but using the equation provided yields:

z = 3.42

By truncating the tolerance limit by 3.42 σ_g, or 3.42 x 0.0002" = 0.000684" instead of 0.0006" on each side, the requirement of 14 ppm maximum rejects is satisfied.

Rounding to 0.0007", the optimal tolerance range for the sorting process is
0.060" +/-0.0023"

A wider tolerance will cause a larger number of rejects in the good bin, while a narrower range will result in increased false rejects.

Of course, the percentage of false rejects needs to be recalculated for the new tolerance range. It will be somewhat larger.

Not centered processes

The same criteria can be applied to a process that produces parts with a mean different from the nominal value of a given dimension.

For example, a 3-sigma process whose mean is offset by +/-1 σ will produce the same number of false rejects as a 2-sigma process, but only on one side, so the total number of false rejects would be approximately half, or 2.19%

Similarly, 30ppm falsely accepted parts can be expected.

Improving the percentage of false rejects

It has been demonstrated that with a reduced tolerance range, only a percentage of the parts in the ambiguity range will be falsely rejected. For a 3-sigma process this number is about 50%; for a 2-sigma process this number is about 66%

This means that the remainder, 50% for a 3-sigma process and 34% for a 2-sigma process will be correctly sorted.

By re-sorting the rejected parts (which should be a reasonably small number), the number of false rejects will be reduced by 50% or 34% each time they are resorted.

As an example consider a lot of 20,000 parts produced by a 2-sigma process:
At an inspection rate of 7,000 parts/hr it would take about 3 hours to generate a set of rejects.

The rejected parts will include those outside of 2 σ (4.5% or 900 parts) plus those falsely rejected (4.2% or 840 parts), totaling 8.7% or 1740 parts.

Re-sorting these rejects would result in the recovery of 34% x 840 parts = 285 parts

There are now 840 – 285 = 555 false rejects.
Re-sorting these rejects would result in the recovery of 34% x 555 parts = 189 parts

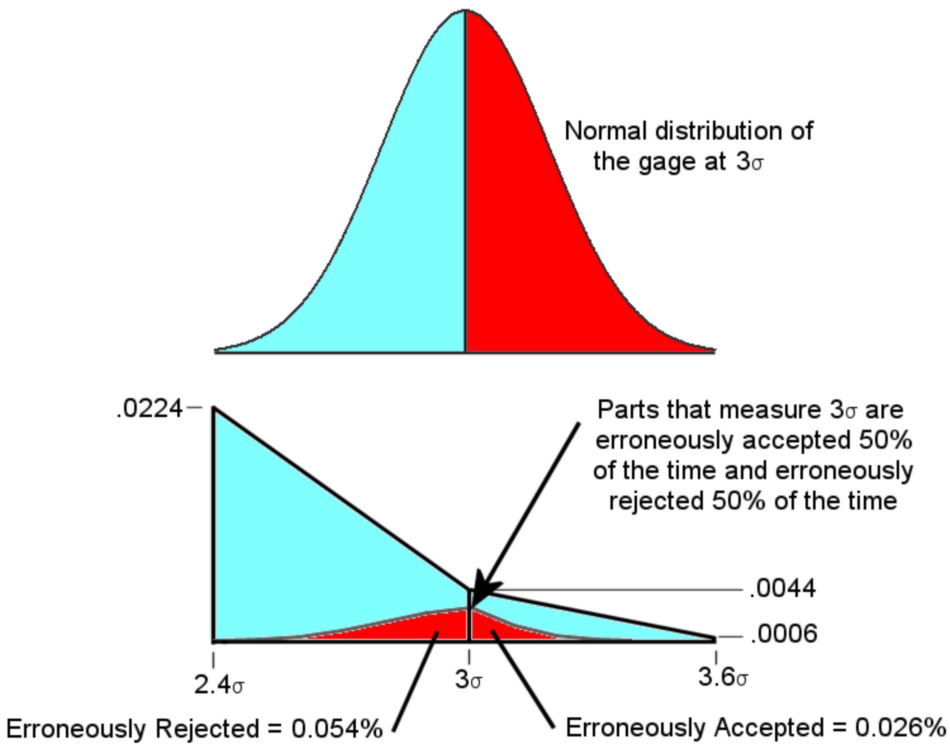
There are now 555 – 189 = 366 false rejects, or just 1.83%

At 7000 parts/hr, it took less than ½ hour (1740 + 1455) to recover 474 parts.

Appendix A

Performance of a sorting process with the tolerance range set the same as the dimension tolerance

Example Process:
Capability: Centered 3-sigma (99%+ product within tolerance)
Dimension: 0.060” +/-.003”
σ_g =0.0002”, or GRR (P/T)= 20%



The table below indicates similar results for various combinations of process capability and gage repeatability. Table entries are x2 to account for the other tolerance limit.

%Erroneously Rejected vs. %Erroneously Accepted

	GRR (P/T) = 10%	GRR (P/T) = 20%	GRR (P/T) = 30%
Equivalent σ_g	$\sigma_g = 0.1\frac{tolerance}{6}$	$\sigma_g = 0.2\frac{tolerance}{6}$	$\sigma_g = 0.3\frac{tolerance}{6}$
3-sigma process	0.043% vs. 0.029%	0.108% vs. 0.052%	0.204% vs. 0.065%
2-sigma process	0.315% vs. 0.265%	0.687% vs. 0.489%	1.125% vs. 0.679%
1-sigma process	0.661% vs. 0.633%	1.350% vs. 1.240%	2.066% vs. 1.820%

Appendix B

Determining the Gage Sigma σ_g

The repeatability of the gage, measured by σ_g, represents the repeatability of the gaging process, and it must include not only the inherent error of the gage but also errors created by the part or measuring process.

As a typical example, consider the use of an optical system such as a comparator or camera based gage used to measure a diameter. This type of instruments will typically measure the diameter at one orientation, as the 3D feature is converted to 2D by the gage. If the diameter is “out-of-round” and measures differently depending on where it’s measured, then these variations would also affect the measurement’s repeatability beyond the repeatability of the gage itself. If the parts are measured while oily, then these variations also need to be included.

The repeatability of the gaging process σ_g, typically needs to be determined empirically as follows:

1. Measure several parts, each part measured repeatedly at random position and orientation, commensurate with the actual process.

2. Calculate the mean of all the measurements on each part, and subtract it from each measurement. This is the centered data, containing just the repeatability information.

3. Calculate the standard deviation of the combined centered data for all parts.

The gaging process repeatability σ_g is directly related to the calculation of the more general GRR, except the limits are not applied. The decision whether to use a given sorting method or not should depend on the acceptance of the cost of false rejects to achieve a tolerable percentage of rejects in the good bin, not the more general %GRR threshold guidelines.

Because some factors created by the manufacturing process may vary with time, this procedure needs to be updated to guarantee the validity of the gaging process repeatability for any sorting application.

Appendix C

Using your sorter to determine the capability of your process.

Once the repeatability of the gage is known, the gage itself may be used to determine the capability of the manufacturing process.

Begin by measuring a dimension on a significant percentage of parts within a given lot, then calculate the mean (μ_c) and standard deviation (σ_c) of all the measurements.

The calculations will represent the convolution of the individual distributions for the process and the gage as follows:

$\mu c = \mu p + \mu g$ and $\sigma c = \sqrt{\sigma p^2 + \sigma g^2}$

Rearranging:

$\mu p = \mu c - \mu g$ and $\sigma p = \sqrt{\sigma c^2 - \sigma g^2}$

Therefore, since we have the convolved parameters μ_c and σ_c, as well as the parameters for the gage μ_g and σ_g, then the parameters for the manufacturing process μ_p and σ_p can easily be calculated.

In many cases, when the standard deviation of the gage is at least 5 times less than the standard deviation of the measurements, then $\sigma_p = \sigma_c$

Because some factors created by the manufacturing process may vary with time, this procedure needs to be updated to guarantee the validity of the manufacturing process capability for any sorting application.